

Formation of Population III Stars in a flat FLRW Universe

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Contrarily to general believe, a first-order cosmological perturbation theory based on Einstein's General Theory of Relativity explains the formation of massive primeval stars in a flat Friedmann-Lemaître-Robertson-Walker universe after decoupling of matter and radiation, whether or not Cold Dark Matter is present. The growth rate of a density perturbation depends on the heat loss of a perturbation during the contraction, but is independent of the particle mass. The relativistic Jeans mass does depend on the particle mass. If the Cold Dark Matter particle mass is equal to the proton mass, then the relativistic Jeans mass is equal to 3500 solar masses, whereas the classical Jeans mass is a factor 145 larger.

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I. INTRODUCTION

A manifestly covariant gauge-invariant cosmological perturbation theory for FLRW universes based on the Theory of General Relativity combined with Thermodynamics and a realistic equation of state for the pressure $p = p(n, \varepsilon)$ has been developed in a foregoing article [1]. In fact, in this article the pioneering work of Lifshitz and Khalatnikov [2, 3] has been redone, but now based on two newly introduced gauge invariant quantities. These quantities, which we baptized $\varepsilon_{(1)}^{\text{gi}}$ and $n_{(1)}^{\text{gi}}$, turned out to be the energy density and the particle number density perturbations. Indeed, taking the non-relativistic limit $v/c \rightarrow 0$ the complete set of relativistic perturbation equations reduce to the complete set of Newtonian equations

$$\nabla^2 \varphi(\mathbf{x}) = 4\pi G \frac{\varepsilon_{(1)}^{\text{gi}}(\mathbf{x})}{c^2}, \quad \varepsilon_{(1)}^{\text{gi}}(\mathbf{x}) = n_{(1)}^{\text{gi}}(\mathbf{x}) m c^2, \quad (1)$$

thus identifying $\varepsilon_{(1)}^{\text{gi}}$ and $n_{(1)}^{\text{gi}}$ as the real energy density and particle number density perturbations.

In the present article it will be demonstrated that the formation of primordial stars, the so-called (hypothetical) population III stars [4, 5], can be understood with the revised Lifshitz-Khalatnikov perturbation theory, whether or not Cold Dark Matter (CDM) is present.

A. Former Results: Standard Perturbation Theory

It is generally accepted that in a universe filled with only 'ordinary matter,' i.e., elementary particles and photons but no CDM, linear perturbation theory predicts a too small growth rate to explain the formation of structure in the universe. The reason put forward in the literature on structure formation is that in the linear phase

of the growth of an *adiabatic* relative density perturbation $\delta(t, \mathbf{x})$ in the era after decoupling of radiation and matter, given by

$$\delta(t) = \delta(t_{\text{dec}}) \left(\frac{t}{t_{\text{dec}}} \right)^{2/3}, \quad t_{\text{dec}} \leq t \leq t_{\text{p}}, \quad (2)$$

is insufficient for relative density perturbations as small as the observed initial value $\delta(t_{\text{dec}}) \approx 10^{-5}$ to reach the non-linear phase for times $t \leq t_{\text{p}}$, where $t_{\text{p}} = 13.75$ Gyr, the present age of the universe, and $t_{\text{dec}} = 381$ kyr, the time of decoupling of matter and radiation [6, 7]. This generally accepted conclusion follows from the standard evolution equation for density perturbations in a universe which is after decoupling of matter and radiation assumed to be filled with a perfect and pressure-less fluid (usually referred to as 'dust') with equations of state for the energy density ε and pressure p

$$\varepsilon = n m c^2, \quad p = 0, \quad (3)$$

where n is the particle number density and m the particle mass.

Before decoupling, Thomson scattering between photons and electrons and Coulomb interactions between electrons and baryons were so rapid that the photons and baryons are tightly coupled so that the photon-baryon system behaves as a single fluid. The standard perturbation theory predicts that density perturbations in this baryon-photon fluid oscillates with a *constant* amplitude and thus do not grow at all before decoupling. Since CDM is supposed to be electrically neutral, it is not linked by Coulomb interactions to the baryon-photon fluid. Therefore, researchers [8, 9] in the field of structure formation have assumed in their simulations that CDM would have already clustered before decoupling and thus would have formed seeds for baryon contraction after decoupling. If this would be true, then the slow growth (2) would be sufficient to explain structure in the universe. Thus, the mechanism of structure formation relies heavily on the particular property of CDM before decoupling, namely

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that density perturbations in CDM are not *electrically* coupled to perturbations in the total energy density.

B. Results from the Revised Lifshitz-Khalatnikov Perturbation Theory

However, in a foregoing article [1] it has been demonstrated that in the radiation-dominated era density perturbations in ordinary matter and CDM are both *gravitationally* coupled to density perturbations in the total energy density. Moreover, it has been found that small-scale density perturbations oscillate in the radiation-dominated era with an *increasing* amplitude, proportional to $t^{1/2}$. These energy (i.e., radiation, ordinary matter and CDM tightly coupled together) density perturbations will form the seeds for star formation after decoupling. They manifest themselves as small temperature fluctuations (24f) in the cosmic background radiation.

Finally, it has been shown that neglecting the kinetic energy density $\frac{3}{2}nk_B T$ with respect to the rest energy density nmc^2 yields in the *perturbed* universe the non-relativistic limit with $\varepsilon_{(1)}^{\text{gr}} = n_{(1)}^{\text{gr}} mc^2$ and $p_{(0)} = p_{(1)}^{\text{gr}} = 0$, (3), implying that $\delta_\varepsilon = \delta_n$. Perturbations described by equations of state (3) are adiabatic. The growth is in this case given by the standard expression (2), which is far too slow to account for structure in the universe. Since adiabatic density perturbations cannot lose their internal energy to their environment, they grow only under the influence of gravitation. This explains their slow growth.

In the *dark ages* of the universe (i.e., the epoch between decoupling and the ignition of the first stars) a density perturbation from which stars will eventually be formed should have initially a somewhat smaller internal pressure than its environment and has to lose some of its heat energy in order to grow faster than given by the standard growth rate (2). It has been established [1] that in a *non-static* universe density perturbations described by an equation of state $p = p(n, \varepsilon)$ are *diabatic*, whereas perturbations described by (3) are adiabatic. Therefore, the evolution of density perturbations should be studied by using a realistic equation of state for the pressure of the form $p = p(n, \varepsilon)$, so that next to gravitational forces, also the heat exchange of a perturbation can be incorporated into the perturbation theory. In fact, incorporating the realistic equations of state (4), yields in the final dynamical perturbation equation (8a) a source term and an evolution equation (8b) for this source term. Using the combined First and Second Laws of Thermodynamics $dE = TdS - pdV + \mu dN$ the source term of equation (8a) can be identified with the entropy of a perturbation. From equation (13) one may infer that the pressure term and the entropy term are of the same order of magnitude. This yields in the early stages of the contraction of a perturbation a somewhat larger growth rate than in the adiabatic case (2). This faster growth is just enough for density perturbations with initial values as small as

$\delta_n \approx \delta_\varepsilon \lesssim 10^{-5}$ to reach the non-linear regime within 10^2 – 10^3 Myr.

II. OUTLINE

Since CDM and ordinary matter particles behave *gravitationally* in exactly the same way and since the mass of a CDM particle is as yet unknown, we assume that the CDM particle mass is approximately equal to the proton mass.

After decoupling the cosmic fluid is a mixture of baryons (protons) and CDM. This mixture can be considered as a non-relativistic monatomic perfect gas with equations of state for the energy density and the pressure

$$\varepsilon(n, T) = nmc^2 + \frac{3}{2}nk_B T, \quad p(n, T) = nk_B T, \quad (4)$$

where k_B is Boltzmann's constant, m the mean particle mass, and T the temperature of the matter. It is assumed that $m = m_H = m_{\text{CDM}}$, where m_H is the proton mass and m_{CDM} the mass of a CDM particle, implying that $mc^2 \gg k_B T$ throughout the matter-dominated era after decoupling. Therefore, one may neglect the pressure $nk_B T$ and kinetic energy density $\frac{3}{2}nk_B T$ with respect to the rest-mass energy density nmc^2 in the *unperturbed* universe: the kinetic energy of the particles in the universe has a negligible influence on the global evolution of the universe. Thus, as is well-known, the global properties of the universe after decoupling are very well described by a perfect and pressure-less fluid ('dust'), described by equations of state (3).

At the moment of decoupling of matter and radiation photons could not ionize matter any more and the two constituents fell out of thermal equilibrium. As a consequence, the pressure drops from a very high radiation pressure $p = \frac{1}{3}a_B T_\gamma^4$ just before decoupling to a very low gas pressure $p = nk_B T$ after decoupling. This fast and chaotic transition from a high pressure epoch to a very low pressure era may result locally in large relative pressure perturbations. These pressure perturbations will be taken into account by incorporating the equations of state (4) and their perturbed counterparts

$$\delta_n - \delta_\varepsilon \approx -\frac{3}{2} \frac{k_B T_{(0)}}{mc^2} \delta_T, \quad \delta_p = \delta_n + \delta_T, \quad (5)$$

into the new perturbation theory. In the expressions (5), $T_{(0)}$ is the background matter temperature and δ_n , δ_ε , δ_T and δ_p are the relative perturbations in the particle number density, the energy density, the matter temperature and the pressure, respectively. The influence of pressure perturbations on the growth of small density perturbations can only be investigated by using the revised Lifshitz-Khalatnikov perturbation theory developed in [1], since this theory not only has an evolution equation (8a) for δ_ε , but (in contrast to all former perturbation theories) also an evolution equation (8b) for the difference $\delta_n - \delta_\varepsilon$. This proves to be crucial for the

understanding of star formation in the early universe. Although in a linear perturbation theory $|\delta_p| \leq 1$ and $|\delta_T| \leq 1$, the initial values of these quantities are, according to (5), not constrained to be as small as the initial values

$$\delta_\varepsilon(t_{\text{dec}}, \mathbf{q}) \approx \delta_n(t_{\text{dec}}, \mathbf{q}) \lesssim 10^{-5}, \quad (6)$$

as is demanded by WMAP-observations [6, 7]. Since the gas pressure $p = nk_B T$ is very low, its relative perturbation $\delta_p \equiv p_{(1)}^{\text{gi}}/p_{(0)}$ and, accordingly, the matter temperature perturbation $\delta_T \equiv T_{(1)}^{\text{gi}}/T_{(0)}$ could be large.

III. RESULTS

Just after decoupling, ordinary matter is mixed with CDM. It is found that the growth rate is independent of the particle mass, i.e., the gravitational mechanism for star formation works equally well with or without CDM.

It will be shown that just after decoupling at $z = 1091$ negative relative matter temperature perturbations as small as -0.5% yields massive stars within 13.75 Gyr. The very first stars, the so-called Population III stars, come into existence between 10^2 Myr and 10^3 Myr. The star masses are in the range from $4 \times 10^2 M_\odot$ to $10^5 M_\odot$, with a peak around $3.5 \times 10^3 M_\odot$. Density perturbations with masses smaller than $3.5 \times 10^3 M_\odot$ become non-linear at later times, because their internal gravity is weaker. On the other hand, density perturbations with masses larger than $3.5 \times 10^3 M_\odot$ enter the non-linear regime also later, since they do not cool down so fast due to their large scale. The mass $3.5 \times 10^3 M_\odot$ corresponds initially to a scale of 6.2 pc. These conclusions are outlined in Figure 1. From this figure it follows that the growth rate rapidly decreases for perturbations with masses below $3.5 \times 10^3 M_\odot$. Therefore, the peak values in Figure 1 can be considered as the relativistic counterparts of the classical *Jeans mass*. However, the Jeans mass does depend on the particle mass: heavier particles yield lighter primordial stars.

IV. BASIC EQUATIONS

For the equations of state (4) the background equations for a flat ($k = 0$) FLRW universe with a vanishing cosmological constant ($\Lambda = 0$) reduce to

$$3H^2 = \kappa \varepsilon_{(0)}, \quad \dot{\varepsilon}_{(0)} = -3H \varepsilon_{(0)}, \quad \dot{n}_{(0)} = -3H n_{(0)}, \quad (7)$$

where it is used that $w \equiv p_{(0)}/\varepsilon_{(0)} \ll 1$, so that the background pressure $p_{(0)}$ can be neglected with respect to the background energy density $\varepsilon_{(0)}$. An overdot denotes differentiation with respect to ct , and $\kappa \equiv 8\pi G/c^4$.

It has been shown in a foregoing article [1] that for equations of state (4) and their perturbed counterparts

(5) the perturbation equations reduce to

$$\ddot{\delta}_\varepsilon + 3H\dot{\delta}_\varepsilon - \left[\beta^2 \frac{\nabla^2}{a^2} + \frac{5}{6} \kappa \varepsilon_{(0)} \right] \delta_\varepsilon = -\frac{2}{3} \frac{\nabla^2}{a^2} (\delta_n - \delta_\varepsilon), \quad (8a)$$

$$\frac{1}{c} \frac{d}{dt} (\delta_n - \delta_\varepsilon) = -2H (\delta_n - \delta_\varepsilon). \quad (8b)$$

The quantity $\beta(t)$ defined by $\beta \equiv \sqrt{\dot{p}_{(0)}/\dot{\varepsilon}_{(0)}}$ is, to a good approximation, given by

$$\beta(t) \approx \frac{v_s(t)}{c} = \sqrt{\frac{5}{3} \frac{k_B T_{(0)}(t)}{m c^2}}, \quad T_{(0)} \propto a^{-2}, \quad (9)$$

with v_s the adiabatic speed of sound and $T_{(0)}$ the matter temperature. Equation (8b) implies

$$\delta_n - \delta_\varepsilon \propto a^{-2}, \quad (10)$$

where it is used that $H \equiv \dot{a}/a$, with $a(t)$ the scale factor of the universe. Combining (9) and (10) one gets from (5)

$$\delta_T(t, \mathbf{x}) \approx \delta_T(t_0, \mathbf{x}), \quad (11)$$

to a very good approximation. Using the well-known solutions of the background equations (7)

$$H \propto t^{-1}, \quad \varepsilon_{(0)} \propto t^{-2}, \quad n_{(0)} \propto t^{-2}, \quad a \propto t^{2/3}, \quad (12)$$

and substituting $\delta(t, \mathbf{x}) = \delta(t, \mathbf{q}) \exp(i\mathbf{q} \cdot \mathbf{x})$, equations (8) can be combined into one equation

$$\delta_\varepsilon'' + \frac{2}{\tau} \delta_\varepsilon' + \left[\frac{4}{9} \frac{\mu_m^2}{\tau^{8/3}} - \frac{10}{9\tau^2} \right] \delta_\varepsilon = -\frac{4}{15} \frac{\mu_m^2}{\tau^{8/3}} \delta_T(t_0, \mathbf{q}), \quad (13)$$

where $\tau \equiv t/t_0$ and a prime denotes differentiation with respect to τ . The parameter μ_m is given by

$$\mu_m \equiv \frac{2\pi}{\lambda_0} \frac{1}{H(t_0)} \frac{v_s(t_0)}{c}, \quad \lambda_0 \equiv \lambda a(t_0), \quad (14)$$

where $\lambda_0 \equiv 2\pi/|\mathbf{q}_0|$ is the scale of a perturbation at time t_0 . The constant μ_m can be expressed in observable quantities. To that end we use that the redshift $z(t)$ as a function of the scale factor $a(t)$ is given by

$$z(t) = \frac{a(t_p)}{a(t)} - 1, \quad (15)$$

where $a(t_p)$ is the present value of the scale factor. For a flat FLRW universe one may take $a(t_p) = 1$. Using (12), (15) and $T_{(0)} \propto a^{-2}$, we get

$$\mu_m = \frac{2\pi}{\lambda_0} \frac{1}{\mathcal{H}(t_p)} \frac{\sqrt{\frac{5}{3} \frac{k_B T_{(0)}(t_{\text{dec}})}{m}}}{[z(t_{\text{dec}}) + 1] \sqrt{z(t_0) + 1}}, \quad (16)$$

where t_0 is the time when a perturbation starts to contract. This expression is invariant under the replacement $m \rightarrow \alpha m$ and $\lambda_0 \rightarrow \lambda_0/\sqrt{\alpha}$, for some constant $\alpha > 0$.

This implies that a perturbation δ_ε with initial scale λ_0 in a cosmic fluid with mean particle mass m , evolves in exactly the same way as a perturbation with initial scale $\lambda_0/\sqrt{\alpha}$ in a fluid with mean particle mass αm . In other words, the growth rate is independent of the particle mass.

The mass $M(t_0)$ of a spherical density perturbation with radius $\frac{1}{2}\lambda_0$ is given by

$$M(t_0) = \frac{4\pi}{3} \left(\frac{1}{2}\lambda_0\right)^3 n_{(0)}(t_0)m. \quad (17)$$

The particle number density $n_{(0)}(t_0)$ can be calculated from its value $n_{(0)}(t_{\text{eq}})$ at the end of the radiation-dominated era. By definition, at the end of the radiation-domination era the matter energy density $n_{(0)}mc^2$ equals the energy density of the radiation:

$$n_{(0)}(t_{\text{eq}})mc^2 = a_B T_{(0)\gamma}^4(t_{\text{eq}}). \quad (18)$$

Since $n_{(0)} \propto a^{-3}$ and $T_{(0)\gamma} \propto a^{-1}$, we find, using (15), the particle number density at time t_0

$$n_{(0)}(t_0) = \frac{a_B T_{(0)\gamma}^4(t_p)}{mc^2} [z(t_{\text{eq}}) + 1] [z(t_0) + 1]^3. \quad (19)$$

Combining (17) and (19), we get for the mass of a spherical density perturbation

$$M(t_0) = \frac{4\pi}{3} \left(\frac{1}{2}\lambda_0\right)^3 \frac{a_B T_{(0)\gamma}^4(t_p)}{c^2} [z(t_{\text{eq}}) + 1] [z(t_0) + 1]^3. \quad (20)$$

With the help of this expression the initial scale λ_0 of a perturbation is related to its mass at the initial time t_0 .

The influence of the mean particle mass m on the mass $M(t_0)$ of a primordial star can be studied by replacing m by αm and λ_0 by $\lambda_0/\sqrt{\alpha}$ ($\alpha > 0$) in expressions (16)–(19). It is found from (20)

$$M(t_0) \propto \alpha^{-3/2}. \quad (21)$$

In other words, the heavier the particles in the universe, the lighter the primordial stars. For example, if $\alpha = 10$ then the mean particle is $m = 10m_H$, implying that the Jeans mass in Figure 1 becomes approximately $10^2 M_\odot$.

Finally, the influence of the initial time on a star mass is determined. It follows from (16) that

$$\lambda_0 \propto [z(t_0) + 1]^{-1/2}, \quad (22)$$

implying with (20) that

$$M(t_0) \propto [z(t_0) + 1]^{3/2}. \quad (23)$$

Thus, the later a perturbation starts to contract, the smaller the mass will be. For example, if a perturbation starts to contract at $z(t_0) = 1$, then the Jeans mass in Figure 1 will be $M_J(t_0) \approx 0.27 M_\odot$.

V. INITIAL VALUES FROM WMAP

The physical quantities measured by WMAP [6, 7] and needed in the present theory of primordial star formation are the redshifts at matter-radiation equality and decoupling, the present values of the Hubble function and the background radiation temperature, the age of the universe and the fluctuations in the background radiation temperature:

$$z(t_{\text{eq}}) = 3196, \quad (24a)$$

$$z(t_{\text{dec}}) = 1091, \quad (24b)$$

$$cH(t_p) = \mathcal{H}(t_p) = 71.0 \text{ km/sec/Mpc}, \quad (24c)$$

$$T_{(0)\gamma}(t_p) = 2.725 \text{ K}, \quad (24d)$$

$$t_p = 13.75 \text{ Gyr}, \quad (24e)$$

$$\delta_{T_\gamma}(t_{\text{dec}}) \lesssim 10^{-5}. \quad (24f)$$

At decoupling the matter temperature is equal to the radiation temperature. The latter can be calculated from the fact that $T_{(0)\gamma} \propto a^{-1}$ and the quantities (24b) and (24d). Using (15), one finds for the matter temperature at decoupling

$$T_{(0)}(t_{\text{dec}}) = T_{(0)\gamma}(t_{\text{dec}}) = 2976 \text{ K}. \quad (25)$$

Substituting the observed values (24b) and (24c) into (16), one gets, using also (25),

$$\mu_m = \frac{518.5}{\lambda_0 \sqrt{z(t_0) + 1}}, \quad \lambda_0 \text{ in pc}, \quad (26)$$

where we have used that $1 \text{ pc} = 3.0857 \times 10^{16} \text{ m}$ ($1 \text{ pc} = 3.2616 \text{ ly}$).

Finally, using that one solar mass is $1.98892 \times 10^{30} \text{ kg}$, we find from (24) that

$$M(t_0) = 1.148 \times 10^{-8} \lambda_0^3 [z(t_0) + 1]^3 M_\odot. \quad (27)$$

The expression (27) will be used to convert the scale λ_0 (expressed in units of 1 pc) of a perturbation, which starts to contract at a redshift of $z(t_0)$, into its mass $M(t_0)$ (expressed in units of the solar mass).

VI. POPULATION III STAR FORMATION

In this section the evolution equation (13) is solved numerically. To that end the differential equation solver **lsodar** with root finding capabilities is used. This solver is included in the package **deSolve** [10], which, in turn, is included in R, a system for statistical computation and graphics [11]. Star formation which starts at cosmological redshift $z = 1091$, i.e., at $t_0 = t_{\text{dec}}$, is investigated.

The WMAP observations of the fluctuations in the background radiation temperature yield for the fluctuations in the energy density and particle number density (6). In addition, it is assumed that

$$\dot{\delta}_\varepsilon(t_{\text{dec}}, \mathbf{q}) \approx 0, \quad (28)$$

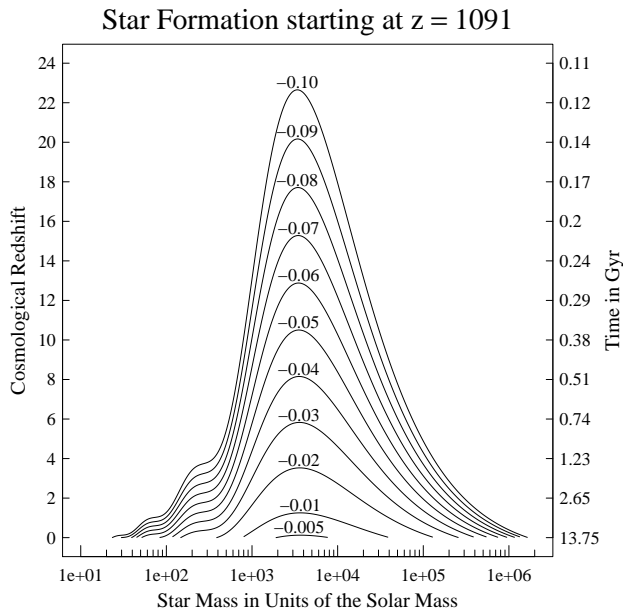


Figure 1. The curves give the redshift at which a linear perturbation in the particle number density with initial values $\delta_n(t_{\text{dec}}, \mathbf{q}) \approx \delta_\varepsilon(t_{\text{dec}}, \mathbf{q}) \approx 10^{-5}$ and $\delta'_n(t_{\text{dec}}, \mathbf{q}) = 0$ starting to grow at an initial redshift of $z(t_{\text{dec}}) = 1091$ becomes non-linear, i.e., $\delta_n \approx \delta_\varepsilon \approx 1$. During the evolution we have $\delta_p(t, \mathbf{q}) = \delta_T(t_{\text{dec}}, \mathbf{q}) + \delta_n(t, \mathbf{q})$. The numbers at each of the curves are the initial relative perturbations in the matter temperature $\delta_T(t_{\text{dec}}, \mathbf{q})$. For each curve, the Jeans mass (i.e., the peak value) is at $3.5 \times 10^3 M_\odot$.

i.e., during the transition from the radiation-dominated era to the era after decoupling energy density perturbations are approximately constant with respect to time.

Figure 1 has been constructed as follows. For each choice of $\delta_T(t_{\text{dec}}, \mathbf{q})$ equation (13) is integrated for a large number of values for the initial scales $\lambda_0 = \lambda_{\text{dec}}$, using the initial values (6) and (28). The integration starts at $\tau \equiv t/t_{\text{dec}} = 1$, i.e., at $z = z(t_{\text{dec}})$ and will be halted if either $z = 0$ (i.e., $\tau = [z(t_{\text{dec}}) + 1]^{3/2}$), or $\delta_\varepsilon(t, \mathbf{q}) = 1$ has been reached. One integration run yields one point on the curve for a particular choice of the scale λ_{dec} if $\delta_\varepsilon(t, \mathbf{q}) = 1$ has been reached for $z > 0$. If the integration halts at $z = 0$ and still $\delta_\varepsilon(t, \mathbf{q}) < 1$, then the perturbation belonging to that particular scale λ_{dec} has not yet reached its non-linear phase today, i.e., at $t_p = 13.75$ Gyr. On the other hand, if the integration is stopped at $\delta_\varepsilon(t, \mathbf{q}) = 1$ and $z > 0$, then the perturbation has become non-linear within 13.75 Gyr.

The above described procedure is repeated for $\delta_T(t_{\text{dec}}, \mathbf{q})$ in the range $-0.005, -0.01, -0.02, \dots, -0.1$. During the evolution, the relative pressure perturbation evolves according to (5) and (11):

$$\delta_p(t, \mathbf{q}) = \delta_T(t_{\text{dec}}, \mathbf{q}) + \delta_n(t, \mathbf{q}). \quad (29)$$

The fastest growth is seen for perturbations with a mass

of approximately $3.5 \times 10^3 M_\odot$. This value is nearly independent of the initial value of the matter temperature perturbation $\delta_T(t_{\text{dec}}, \mathbf{q})$. Even density perturbations with an initial relative matter temperature perturbation as small as $\delta_T(t_{\text{dec}}, \mathbf{q}) = -0.5\%$ reach their non-linear phase at $z = 0.13$ ($T_{(0)\gamma} = 3.1$ K, $t = 11.5$ Gyr) provided that its mass is around $3.5 \times 10^3 M_\odot$. Perturbations with masses smaller than $3.5 \times 10^3 M_\odot$ reach their non-linear phase at a later time, because their internal gravity is weaker. On the other hand, perturbations with masses larger than $3.5 \times 10^3 M_\odot$ cool down slower because of their large scales, resulting also in a smaller growth rate. Since the growth rate decreases rapidly for perturbations with masses below $3.5 \times 10^3 M_\odot$, the latter mass will be considered as the relativistic counterpart of the classical *Jeans mass*. This mass corresponds to a Jeans scale of $6.2 \text{ pc} \approx 20 \text{ ly}$. This scale is much smaller than the horizon size at decoupling, given by $d_H(t_{\text{dec}}) = 3ct_{\text{dec}} \approx 350 \text{ kpc}$.

VII. HEAT LOSS DURING CONTRACTION

In this section the heat loss of a density perturbation during its contraction is calculated. To that end the combined first second law of thermodynamics (24) in Ref. [1] is rewritten in the form

$$T_{(0)} s_{(1)}^{\text{gi}} = -\frac{\varepsilon_{(0)}}{n_{(0)}} (\delta_n - \delta_\varepsilon) - \frac{p_{(0)}}{n_{(0)}} \delta_n, \quad (30)$$

where it is used that $w \equiv p_{(0)}/\varepsilon_{(0)}$. Substituting expressions (4) and (5) into (30) and using also (11), one finds the entropy per particle of a density perturbation:

$$s_{(1)}^{\text{gi}}(t, \mathbf{x}) \approx \frac{1}{2} k_B [3\delta_T(t_0, \mathbf{x}) - 2\delta_n(t, \mathbf{x})], \quad (31)$$

where it is used that $mc^2 \gg k_B T_{(0)}$. For all values of $\delta_T(t_0, \mathbf{x})$ in Figure 1 and initial values (6) the entropy perturbation is negative, $s_{(1)}^{\text{gi}} < 0$. Since for growing perturbations one has $\dot{\delta}_n > 0$ the entropy perturbation decreases, i.e., $\dot{s}_{(1)}^{\text{gi}} = -k_B \dot{\delta}_n < 0$, during contraction. This implies that a growing perturbation loses a part of its internal energy to its environment. This is to be expected, since a local density perturbation is not isolated from its environment. Only for an isolated system the entropy never decreases.

VIII. CLASSICAL JEANS MASS

In this section the classical Jeans mass, derived from the Newtonian theory of gravity, is compared with the relativistic Jeans mass which follows from the revised Lifshitz-Khalatnikov perturbation theory.

The classical Jeans length at time t_0 is given by [12]

$$\lambda_J(t_0) = v_s(t_0) \sqrt{\frac{\pi}{G n_{(0)}(t_0) m}}. \quad (32)$$

Using (9), (17) and (19) one finds for the classical Jeans mass at decoupling:

$$M_J(t_{\text{dec}}) \approx 5.1 \times 10^5 M_\odot, \quad (33)$$

where the WMAP values (24) have been used. The classical Jeans mass (33) is much larger than the relativistic Jeans mass, $3.5 \times 10^3 M_\odot$ which follows from the revised Lifshitz-Khalatnikov perturbation theory. This difference is due to the fact that in the classical perturbation theory based on the equations of state (3) the heat loss of a perturbation is not taken into account, whereas the effect of heat loss on the growth of a perturbation is included in the revised Lifshitz-Khalatnikov perturbation theory based on the equations of state (4). In other words, since a perturbation loses some of its energy, gravity can be somewhat weaker to make a perturbation contract. The classical Jeans mass (33) corresponds to a classical Jeans scale of $32.4 \text{ pc} \approx 106 \text{ ly}$. Just as is the case for μ_m (16), the expression (32) is invariant under the replacement $m \rightarrow \alpha m$ and $\lambda_J \rightarrow \lambda_J/\sqrt{\alpha}$, for some constant $\alpha > 0$. As a consequence, the classical Jeans mass M_J is proportional to $\alpha^{-3/2}$, just as in the relativistic case (21).

Finally, the classical Jeans mass of a perturbation starting at $z(t_0) = 1$ follows from (23) and (33). It is

found that $M_J(t_0) \approx 40 M_\odot$.

IX. CONCLUSION

Three important conclusions can now be drawn. Firstly, there is no need to make use of alternative gravitational theories: the Theory of General Relativity explains the formation of massive primordial stars in our universe. In other words, Einstein's gravitational theory not only describes the *global* characteristics of the universe, but is also *locally* successful. Secondly, although there is strong evidence for the existence of CDM [13], it is not needed for the formation of primeval stars. Finally, it has been demonstrated that not only for large-scale perturbations one should use the theory of relativity, but also for small-scale perturbations: because of the spurious gauge modes present in the Newtonian theory of gravity, it fails to predict primordial stars.

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